Math 10B with Professor Stankova
Worksheet, Discussion #6; Tuesday, 2/6/2018
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Algorithms

Examples

1. Demonstrate bubble sort to sort the list 3, 4, 2, 1.

Solution: On the first pass, we go 3421 → 3241 → 3214, on the next pass we do 3214 → 2314 → 2134 and on the third pass we do 2134 → 1234 and on the fourth pass, we make no changes which means the algorithm terminates.

2. Demonstrate the quick sort to sort the list $3, 6, 2, 5, 1, 4$.

Solution: First we place the first number in the appropriate position and put the numbers smaller before it and the numbers larger after while preserving their relative order to get 362514 → 213654. Now we do the same on the smaller numbers and the larger to get 21 → 12 and 654 → 546 → 456. This finally sorts the list as 123456.

3. Demonstrate the stable matching algorithm when men and women have the preferences $m_1 : w_1 > w_2, m_2 : w_1 > w_2$ and $w_1 : m_1 > m_2, w_2 : m_1 > m_2$.

Solution: Both men will propose to woman 1 and she will choose man number 1. Then man 2 will propose to his next option which is woman 2 and she will accept him. Thus, we get the final pairing $(m_1, w_1), (m_2, w_2)$.

Problems

4. **TRUE** False The stable matching algorithm with always produce a matching that is stable.
5. True **FALSE** There is only one stable matching.

Induction

Examples

6. Prove that $1 + 2 + \cdots + n = \frac{n(n+1)}{2}$.

Solution: First we show the base case of $n = 1$. In that case, we have that $1 = \frac{1(2)}{2}$ as required. Now assume the inductive hypothesis $S_n : 1 + 2 + \cdots + n = \frac{n(n+1)}{2}$. We wish to prove that $S_{n+1} : 1 + 2 + \cdots + (n + 1) = \frac{(n+1)(n+2)}{2}$. By the inductive hypothesis, we have that

$$(1+2+\cdots+n)+(n+1) = \frac{n(n + 1)}{2}+(n+1) = \frac{n(n + 1) + 2(n + 1)}{2} = \frac{(n + 2)(n + 1)}{2}.$$

Therefore, by the principle of mathematical induction, we have proven the result.